

Appl. No. 10/605,250  
Amdt. dated June 20, 2007  
Reply to Office action of April 05, 2007

Amendments to the Drawings:

Four replacement sheets are attached having Figures 4-7 thereon.

Figures 4 and 6 have been amended to change the spelling of "vawe" to become "value".

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Figures 5 and 7 have been amended to change the functions  $h(s)$ ,  $h_1(s)$ ,  $h_2(s)$ , and  $h_3(s)$  to become  $h(x,y)$ ,  $h_1(x,y)$ ,  $h_2(x,y)$ , and  $h_3(s)$ , respectively.

In addition, step 112 of Figure 6 has been amended to move the comma at the beginning  
10 of the second line to the end of the first line.

Acceptance of the corrected drawings is respectfully requested.

Attachment: Replacement Sheets

4 pages

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**REMARKS/ARGUMENTS**

1. Request for information under 37 CFR 1.105:

Applicant and the assignee of this application are required under 37 CFR 1.105 to provide information that is reasonably necessary to the examination of this application:

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**Response:**

According to the applicant's records, a prior art search was performed using the search strings of "abstract/(image and lowpass and filter and sharp\$)" to perform a search in the USPTO search database. The resulting prior art US patents that matched this search 10 were US 5,189,511, US 5,428,456, and US 6,411,305. The records for this application do not show any of the inventor's comments regarding these prior art patents.

In an internal invention disclosure proposed by the inventor, the reference, " R. V. Hogg & A. T. Craig, "Introduction to Mathematical Statistics," 5th edition, Prentice Hall, 15 pp147, 1995." is listed as a reference of the application. The content of the reference is attached herewith.

2. Objection to the specification:

The specification is objected to due to numerous informalities. Appropriate 20 correction is required.

**Response:**

Upon inspection of this application, the applicant has noticed significant differences between the copy of the specification on the USPTO's Public PAIR website and the 25 Patent Application Publication 2005/0008249 of this application.

There seems to have been problems in printing this application, and various errors

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have been introduced through the printing process. Virtually every error mentioned by the Examiner in the office action was caused through the printing process. To help correct this process, and for convenience of examination, the applicant is providing a substitute specification along with a listing of the claims. No amendments have been made to the 5 specification.

Regarding the specification, the applicant has not noticed any instances of the term  $I(x,y)$  being replaced by  $I(x,y)$ . In addition, the symbol for convolution is used consistently in both the specification and the drawings. Acceptance of the specification is 10 respectfully requested.

**3. Rejection of claims 1-20 under 35 U.S.C. 112, first paragraph:**

Claims 1-20 are rejected under 35 U.S.C. 112, first paragraph, as failing to comply with the enablement requirement.

15 Claim 1 recites "generating an energy ratio of a band-pass image signal and the image signal according to the standard deviation of a first low pass signal of the first low pass filter and a second low pass signal of the second low pass filter". The energy ratio as defined in paragraph 49 of the application is a function of the image signal, yet paragraph 20 54 of the specification equates the energy ratio to a function that is independent of the image signal.

Claim 11 has the same problem.

25 **Response:**

As shown in paragraphs [0093] to [0113] of Patent Application Publication

2005/0008249 (particularly paragraph [0098]), the energy ratio  $\frac{E_{\text{bpl}}}{E_s}$  is shown to be

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equivalent to  $\frac{1}{4\pi\sigma_1^2} - \frac{2}{2\pi(\sigma_1^2 + \sigma_2^2)} + \frac{1}{4\pi\sigma_2^2}$ . Thus, one skilled in the art would be able to understand the two equations presented in paragraphs [0049] and [0054] from reading this later section of the specification.

5 The terms  $\sigma_1$  and  $\sigma_2$  represent respectively the standard deviations of the low pass filter signal  $h_1(x, y)$  of the first low pass filter 38 and the low pass filter signal  $h_2(x, y)$  of the second low pass filter 40.

10 In view of the above, the applicant believes that claims 1-20 comply with the enablement requirement. Reconsideration of claims 1-20 is respectfully requested.

4. Rejection of claims 1-20 under 35 U.S.C. 101:

Claims 1-20 are rejected under 35 U.S.C. 101 because the claimed invention is directed to non-statutory subject matter.

15 Claims 1 and 11 recite a mathematical function that is not limited to a practical application.

**Response:**

20 Claims 1 and 11 have been amended to recite the step of storing the adjusted image signal in a computer-readable memory. This amendment is fully supported in Figure 3 along with paragraph [0025] of the specification. Since the adjusted image signal is stored in a computer-readable memory, a tangible result is produced by the methods of claims 1 and 11. In view of the claim amendments, reconsideration of claims 1-20 is therefore requested.

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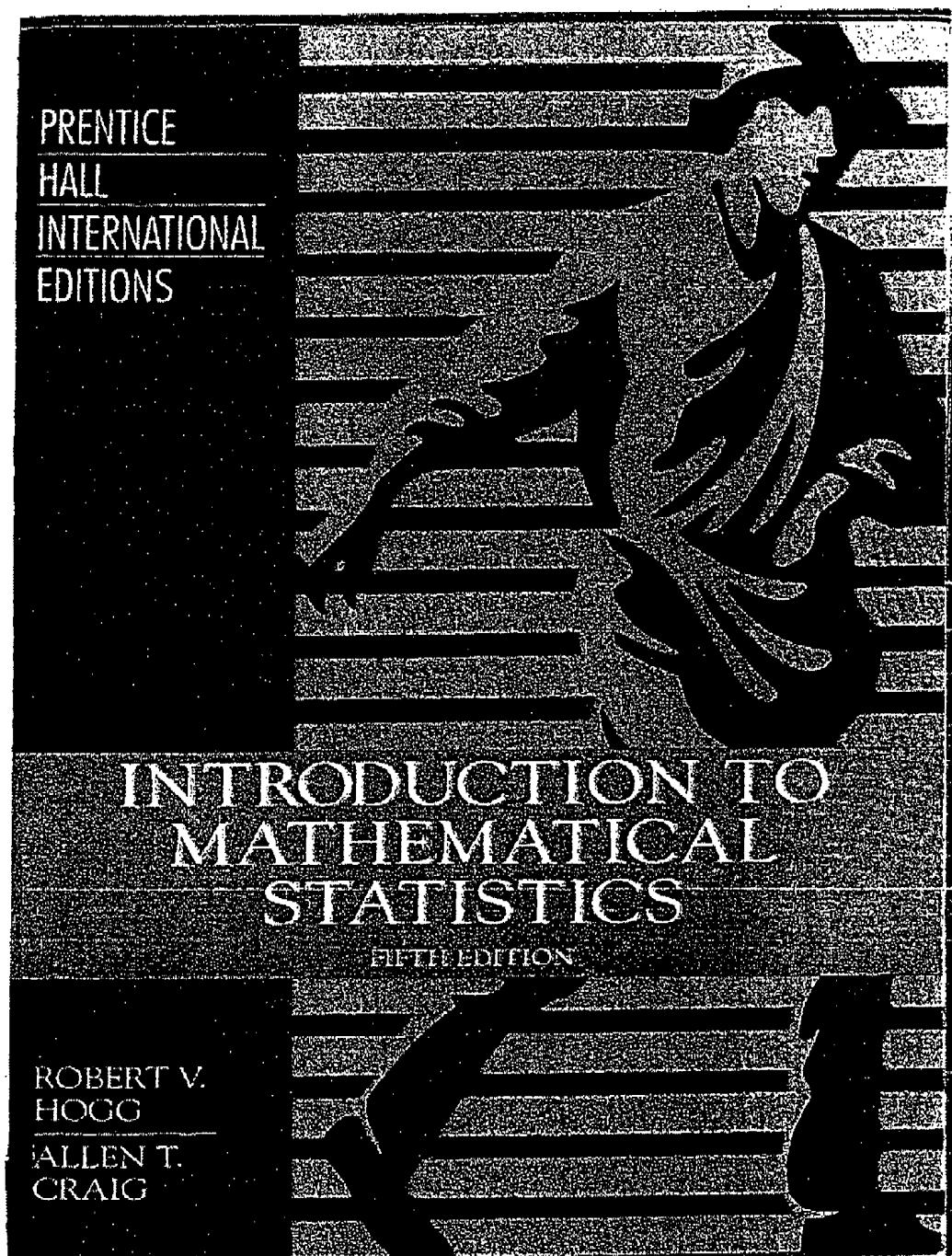
Applicant respectfully requests that a timely Notice of Allowance be issued in this case.

Sincerely yours,

5 Winston Hsu Date: 06.20.2007

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Note: Please leave a message in my voice mail if you need to talk to me. (The time in D.C.  
is 12 hours behind the Taiwan time, i.e. 9 AM in D.C. = 9 PM in Taiwan.)



## Some Special Distributions [Ch. 3]

## 146 Sec. 3.5: The Bivariate Normal Distribution 147

3.64. Compare the measures of skewness and kurtosis of a distribution which is  $N(\mu, \sigma^2)$ .

3.65. Let the random variable  $X$  have a distribution that is  $N(\mu, \sigma^2)$ .

- (a) Does the random variable  $Y = X^2$  also have a normal distribution?
- (b) Would the random variable  $Y = aX + b$ ,  $a$  and  $b$  nonzero constants, have a normal distribution?

*Hint:* In each case, first determine  $\Pr(Y \leq y)$ .

3.66. Let the random variable  $X$  be  $N(\mu, \sigma^2)$ . What would this distribution be if  $\sigma^2 = 0$ ?

*Hint:* Look at the m.g.f. of  $X$  for  $\sigma^2 > 0$  and investigate its limit as  $\sigma^2 \rightarrow 0$ .

3.67. Let  $\varphi(x)$  and  $\Phi(x)$  be the p.d.f. and distribution function of a standard normal distribution. Let  $Y$  have a truncated distribution with p.d.f.  $g(y) = [\varphi(a) - \varphi(b)]/\Phi(b) - \Phi(a)$ ,  $a < y < b$ , zero elsewhere. Show that  $E(Y)$  is equal to  $[\varphi(a) - \varphi(b)]/\Phi(b) - \Phi(a)$ .

3.68. Let  $f(x)$  and  $F(x)$  be the p.d.f. and the distribution function of a distribution of the continuous type such that  $f(x)$  exists for all  $x$ . Let the mean of the truncated distribution that has p.d.f.  $g(y) = f(y)/F(b)$ ,  $-\infty < y < b$ , zero elsewhere, be equal to  $-f(b)/F(b)$  for all real  $b$ . Prove that  $f(x)$  is a p.d.f. of a standard normal distribution.

3.69. Let  $X$  and  $Y$  be independent random variables, each with a distribution that is  $N(0, 1)$ . Let  $Z = X + Y$ . Find the integral that represents the distribution function  $G(z) = \Pr(X + Y \leq z)$  of  $Z$ . Determine the p.d.f. of  $Z$ .

*Hint:* We have that  $G(z) = \int_{-\infty}^z H(x, z) dx$ , where

$$H(x, z) = \int_{-\infty}^{x+z} \frac{1}{2\pi} \exp[-(x^2 + y^2)/2] dy.$$

Find  $G'(z)$  by evaluating  $\int_{-\infty}^{\infty} |\partial H(x, z)/\partial z| dx$ .

### 3.5 The Bivariate Normal Distribution

**Remark.** If the reader with an adequate background in matrix algebra so chooses, this section can be omitted at this point and Section 4.10 can be considered later. If this decision is made, only an example in Section 4.7 and a few exercises need be skipped because the bivariate normal distribution would not be known. Many statisticians, however, find it easier to remember the multivariate (including the bivariate) normal p.d.f. and m.g.f. using matrix notation that is used in Section 4.10. Moreover, that section provides an excellent example of a transformation (in particular, an orthogonal one)

and a good illustration of the moment-generating function technique; these are two of the major concepts introduced in Chapter 4.

Let us investigate the function

$$f(x, y) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} e^{-\eta x^2}, \quad -\infty < x < \infty, \quad -\infty < y < \infty,$$

where, with  $\sigma_1 > 0$ ,  $\sigma_2 > 0$ , and  $-1 < \rho < 1$ ,

$$\eta = \frac{1}{1-\rho^2} \left[ \left( \frac{x-\mu_1}{\sigma_1} \right)^2 - 2\rho \left( \frac{x-\mu_1}{\sigma_1} \right) \left( \frac{y-\mu_2}{\sigma_2} \right) + \left( \frac{y-\mu_2}{\sigma_2} \right)^2 \right].$$

At this point we do not know that the constants  $\mu_1$ ,  $\mu_2$ ,  $\sigma_1^2$ ,  $\sigma_2^2$ , and  $\rho$  are those respective parameters of a distribution. As a matter of fact, we do not know that  $f(x, y)$  has the properties of a joint p.d.f. It will be shown that:

1.  $f(x, y)$  is a joint p.d.f.
2.  $X$  is  $N(\mu_1, \sigma_1^2)$  and  $Y$  is  $N(\mu_2, \sigma_2^2)$ .
3.  $\rho$  is the correlation coefficient of  $X$  and  $Y$ .

A joint p.d.f. of this form is called a *bivariate normal p.d.f.*, and the random variables  $X$  and  $Y$  are said to have a *bivariate normal distribution*.

That the nonnegative function  $f(x, y)$  is actually a joint p.d.f. can be seen as follows. Define  $f_1(x)$  by

$$f_1(x) = \int_{-\infty}^{\infty} f(x, y) dy.$$

Now

$$(1-\rho^2)q = \left[ \left( \frac{y-\mu_2}{\sigma_2} \right)^2 - \rho \left( \frac{x-\mu_1}{\sigma_1} \right) \left( \frac{y-\mu_2}{\sigma_2} \right) + (1-\rho^2) \left( \frac{x-\mu_1}{\sigma_1} \right)^2 \right]$$

$$= \left( \frac{y-\mu_2}{\sigma_2} \right)^2 + (1-\rho^2) \left( \frac{x-\mu_1}{\sigma_1} \right)^2,$$

where  $b = \mu_2 + \rho(\sigma_2/\sigma_1)(x - \mu_1)$ . Thus

$$f_1(x) = \frac{\exp \left[ -(x-\mu_1)^2/2\sigma_1^2 \right]}{\sigma_1 \sqrt{2\pi}} \frac{\exp \left\{ -(y-b)^2/[2\sigma_2^2(1-\rho^2)] \right\}}{\sigma_2 \sqrt{1-\rho^2} \sqrt{2\pi}} dy.$$

For the purpose of integration, the integrand of the integral in this

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## Sec. 3.5] The Bivariate Normal Distribution

## Some Special Distributions [Ch. 3

Accordingly,  $M(t_1, t_2)$  can be written in the form

$$\exp \left\{ t_1 \mu_1 - t_2 \rho \frac{\sigma_2}{\sigma_1} \mu_1 + \frac{t_1^2 \sigma_1^2 (1 - \rho^2)}{2} \right\} \int_{-\infty}^{\infty} \exp \left[ \left( t_1 + t_2 \rho \frac{\sigma_2}{\sigma_1} \right) x \right] f_1(x) dx.$$

But  $E(e^{tX}) = \exp [\mu_1 t + (\sigma_1^2 t^2)/2]$  for all real values of  $t$ . Accordingly, if we set  $t = t_1 + t_2 \rho (\sigma_2/\sigma_1)$ , we see that  $M(t_1, t_2)$  is given by

$$\exp \left\{ t_1 \mu_1 - t_2 \rho \frac{\sigma_2}{\sigma_1} \mu_1 + \frac{t_1^2 \sigma_1^2 (1 - \rho^2)}{2} + \mu_1 \left( t_1 + t_2 \rho \frac{\sigma_2}{\sigma_1} \right)^2 + \sigma_1^2 \frac{\left( t_1 + t_2 \rho \frac{\sigma_2}{\sigma_1} \right)^2}{2} \right\}$$

or, equivalently,

$$M(t_1, t_2) = \exp \left( \mu_1 t_1 + \mu_2 t_2 + \frac{\sigma_1^2 t_1^2 + 2\rho\sigma_1\sigma_2 t_1 t_2 + \sigma_2^2 t_2^2}{2} \right).$$

It is interesting to note that if, in this m.g.f.  $M(t_1, t_2)$ , the correlation coefficient  $\rho$  is set equal to zero, then

$$M(t_1, t_2) = M(t_1, 0)M(0, t_2).$$

Thus  $X$  and  $Y$  are independent when  $\rho = 0$ . If, conversely,  
 $M(t_1, t_2) \equiv M(t_1, 0)M(0, t_2)$ ,

we have  $\rho = \sigma_1 \sigma_2 / \sigma_1 \sigma_2 = 1$ . Since each of  $\sigma_1$  and  $\sigma_2$  is positive, then  $\rho = 0$ . Accordingly, we have the following theorem.

**Theorem 3.** Let  $X$  and  $Y$  have a bivariate normal distribution with means  $\mu_1$  and  $\mu_2$ , positive variances  $\sigma_1^2$  and  $\sigma_2^2$ , and correlation coefficient  $\rho$ . Then  $X$  and  $Y$  are independent if and only if  $\rho = 0$ .

As a matter of fact, if any two random variables are independent and have positive standard deviations, we have noted in Example 4 of Section 2.4 that  $\rho = 0$ . However,  $\rho = 0$  does not in general imply that two variables are independent; this can be seen in Exercises 2.20(c) and 2.25. The importance of Theorem 3 lies in the fact that we now know when and only when two random variables that have a bivariate normal distribution are independent.

## EXERCISES

3.70. Let  $X$  and  $Y$  have a bivariate normal distribution with respective parameters  $\mu_X = 2.8$ ,  $\mu_Y = 11.0$ ,  $\sigma_X^2 = 0.16$ ,  $\sigma_Y^2 = 100$ , and  $\rho = 0.6$ . Compute:

- (a)  $\Pr(106 < Y < 124)$ ,
- (b)  $\Pr(106 < Y < 124 | X = 3.2)$ .

3.71. Let  $X$  and  $Y$  have a bivariate normal distribution with parameters  $\mu_1 = 3$ ,  $\mu_2 = 1$ ,  $\sigma_1^2 = 16$ ,  $\sigma_2^2 = 25$ , and  $\rho = \frac{3}{4}$ . Determine the following probabilities:

- (a)  $\Pr(3 < Y < 8)$ ,
- (b)  $\Pr(3 < Y < 8 | X = 7)$ ,
- (c)  $\Pr(-3 < X < 3)$ ,
- (d)  $\Pr(-3 < X < 3 | Y = -4)$ .

3.72. If  $M(t_1, t_2)$  is the m.g.f. of a bivariate normal distribution, compute the covariance by using the formula

$$\frac{\partial^2 M(0, 0)}{\partial t_1 \partial t_2} = \frac{\partial M(0, 0)}{\partial t_1} \frac{\partial M(0, 0)}{\partial t_2}.$$

Now let  $\psi(t_1, t_2) = \ln M(t_1, t_2)$ . Show that  $\partial^2 \psi(0, 0) / \partial t_1 \partial t_2$  gives this covariance directly.

3.73. Let  $X$  and  $Y$  have a bivariate normal distribution with parameters  $\mu_1 = 5$ ,  $\mu_2 = 10$ ,  $\sigma_1^2 = 1$ ,  $\sigma_2^2 = 25$ , and  $\rho > 0$ . If  $\Pr(4 < Y < 16 | X = 5) = 0.954$ , determine  $\rho$ .

3.74. Let  $X$  and  $Y$  have a bivariate normal distribution with parameters  $\mu_1 = 20$ ,  $\mu_2 = 40$ ,  $\sigma_1^2 = 9$ ,  $\sigma_2^2 = 4$ , and  $\rho = 0.6$ . Find the shortest interval for which 0.90 is the conditional probability that  $Y$  is in this interval, given that  $X = 22$ .

3.75. Say the correlation coefficient between the heights of husbands and wives is 0.70 and the mean male height is 5 feet 10 inches with standard deviation 2 inches, and the mean female height is 5 feet 4 inches with standard deviation  $\frac{1}{2}$  inches. Assuming a bivariate normal distribution, what is the best guess of the height of a woman whose husband's height is 6 feet? Find a 95 percent prediction interval for her height.

3.76. Let

$$f(x, y) = (1/2\pi) \exp \{ -(x^2 + y^2) / 2 \} [1 + xy \exp \{ -\frac{1}{2}(x^2 + y^2 - 2)\}],$$

where  $-\infty < x < \infty$ ,  $-\infty < y < \infty$ . If  $f(x, y)$  is a joint p.d.f., it is not a normal bivariate p.d.f. Show that  $f(x, y)$  actually is a joint p.d.f. and that each marginal p.d.f. is normal. Thus the fact that each marginal p.d.f. is normal does not imply that the joint p.d.f. is bivariate normal.

3.77. Let  $X$ ,  $Y$ , and  $Z$  have the joint p.d.f.

$$\left(\frac{1}{2\pi}\right)^{3/2} \exp\left(-\frac{x^2+y^2+z^2}{2}\right) \left[1 + xyz \exp\left(-\frac{x^2+y^2+z^2}{2}\right)\right],$$

where  $-\infty < x < \infty$ ,  $-\infty < y < \infty$ , and  $-\infty < z < \infty$ . While  $X$ ,  $Y$ , and  $Z$  are obviously dependent, show that  $X$ ,  $Y$ , and  $Z$  are pairwise independent and that each pair has a bivariate normal distribution.

3.78. Let  $X$  and  $Y$  have a bivariate normal distribution with parameters  $\mu_1 = \mu_2 = 0$ ,  $\sigma_1^2 = \sigma_2^2 = 1$ , and correlation coefficient  $\rho$ . Find the distribution of the random variable  $Z = ax + by$  in which  $a$  and  $b$  are nonzero constants.

*Hint:* Write  $G(z) = \Pr(Z \leq z)$  as an iterated integral and compute  $G'(z) = g(z)$  by differentiating under the first integral sign and then evaluating the resulting integral by completing the square in the exponent.

#### ADDITIONAL EXERCISES

3.79. Let  $X$  have a binomial distribution with parameters  $n = 288$  and  $p = \frac{1}{3}$ . Use Chebyshev's inequality to determine a lower bound for  $\Pr(76 < X < 116)$ .

3.80. Let  $f(x) = \frac{e^{-x}x^r}{r!}$ ,  $x = 0, 1, 2, \dots$ , zero elsewhere. Find the values of  $\mu$  so that  $x = 1$  is the unique mode; that is,  $f(0) < f(1)$  and  $f(1) > f(2) > f(3) > \dots$

3.81. Let  $X$  and  $Y$  be two independent binomial variables with parameters  $n = 4$ ,  $p = \frac{1}{2}$  and  $n = 3$ ,  $p = \frac{1}{3}$ , respectively. Determine  $\Pr(X - Y = 3)$ .

3.82. Let  $X$  and  $Y$  be two independent binomial variables, both with parameters  $n$  and  $p = \frac{1}{2}$ . Show that

$$\Pr(X - Y = 0) = \frac{(2n)!}{n! n! (2^n)}.$$

3.83. Two people toss a coin five independent times each. Find the probability that they will obtain the same number of heads.

3.84. Color blindness appears in 1 percent of the people in a certain population. How large must a sample with replacement be if the probability of its containing at least one color-blind person is to be at least 0.99? Assume a binomial distribution  $b(n, p = 0.01)$  and find  $n$ .

3.85. Assume that the number  $X$  of hours of sunshine per day in a certain place has a chi-square distribution with 10 degrees of freedom. The profit

of a certain outdoor activity depends upon the number of hours of sunshine through the formula

$$\text{Profit} = 1000(1 - e^{-X/10}).$$

Find the expected level of the profit.

3.86. Place five similar balls (each either red or blue) in a bowl at random as follows: A coin is flipped 5 independent times and a red ball is placed in the bowl for each head and a blue ball for each tail. The bowl is then taken and two balls are selected at random without replacement. Given that each of those two balls is red, compute the conditional probability that 5 red balls were placed in the bowl at random.

3.87. If a die is rolled four independent times, what is the probability of one four, two fives, and one six, given that at least one six is produced?

3.88. Let the p.d.f.  $f(x)$  be positive on, and only on, the integers  $0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$ , so that  $f(x) = [(11 - x)/x]f(x - 1)$ ,  $x = 1, 2, 3, \dots, 10$ . Find  $f(x)$ .

3.89. Let  $X$  and  $Y$  have a bivariate normal distribution with  $\mu_1 = 5$ ,  $\mu_2 = 10$ ,  $\sigma_1^2 = 1$ ,  $\sigma_2^2 = 25$ , and  $\rho = \frac{1}{2}$ . Compute  $\Pr(7 < Y < 19 | x = 5)$ .

3.90. Say that Jim has three cents and that Bill has seven cents. A coin is tossed ten independent times. For each head that appears, Bill pays Jim two cents, and for each tail that appears, Jim pays Bill one cent. What is the probability that neither person is in debt after the ten trials?

3.91. If  $E(X') = [(r+1)N/2]$ ,  $r = 1, 2, 3, \dots$ , find the m.g.f. and p.d.f. of  $X$ .

3.92. For a biased coin, say that the probability of exactly two heads in three independent tosses is  $\frac{1}{5}$ . What is the probability of exactly six heads in nine independent tosses of this coin?

3.93. It is discovered that 75 percent of the pages of a certain book contain no errors. If we assume that the number of errors per page follows a Poisson distribution, find the percentage of pages that have exactly one error.

3.94. Let  $X$  have a Poisson distribution with double mode at  $x = 1$  and  $x = 2$ . Find  $\Pr[X = 0]$ .

3.95. Let  $X$  and  $Y$  be jointly normally distributed with  $\mu_x = 20$ ,  $\mu_y = 40$ ,  $\sigma_x = 3$ ,  $\sigma_y = 2$ ,  $\rho = 0.6$ . Find a symmetric interval about the conditional mean, so that the probability is 0.90 that  $Y$  lies in that interval given that  $X$  equals 25.

3.96. Let  $f(x) = \begin{cases} 10 & p^x(1-p)^{10-x}, x = 0, 1, \dots, 10, \\ 0 & \text{elsewhere.}\end{cases}$  Find the values of  $p$ , so that  $f(0) \geq f(1) \geq \dots \geq f(10)$ .

3.97. Let  $f(x, y)$  be a bivariate normal p.d.f. and let  $c$  be a positive constant so that  $c < (2\pi\sigma_1\sigma_2\sqrt{1-\rho^2})^{-1}$ . Show that  $c = f(x, y)$  defines an ellipse in the  $xy$ -plane.

3.98. Let  $f_1(x, y)$  and  $f_2(x, y)$  be two bivariate normal probability density functions, each having means equal to zero and variances equal to 1. The respective correlation coefficients are  $\rho$  and  $-\rho$ . Consider the joint distribution of  $X$  and  $Y$  defined by the joint p.d.f.  $f_1(x, y) + f_2(x, y)/2$ . Show that the two marginal distributions are both  $N(0, 1)$ ,  $X$  and  $Y$  are dependent, and  $E(XY) = 0$  and hence the correlation coefficient of  $X$  and  $Y$  is zero.

3.99. Let  $X$  be  $N(\mu, \sigma^2)$ . Define the random variable  $Y = e^X$  and find its p.d.f. by differentiating  $G(y) = \Pr(e^X \leq y) = \Pr(X \leq \ln y)$ . This is the p.d.f. of a lognormal distribution.

3.100. In the proof of Theorem 1 of Section 3.4, we could let

$$G(w) = \Pr(X \leq w\sigma + \mu) = F(w\sigma + \mu),$$

where  $F$  and  $F' = f$  are the distribution function and p.d.f. of  $X$ , respectively. Then, by the chain rule,

$$g(w) = G'(w) = [F'(w\sigma + \mu)]\sigma.$$

Show that the right-hand member is the p.d.f. of a standard normal distribution, thus this provides another proof of Theorem 1.

## CHAPTER 4

# Distributions of Functions of Random Variables

### 4.1 Sampling Theory

Let  $X_1, X_2, \dots, X_n$  denote  $n$  random variables that have the joint p.d.f.  $f(x_1, x_2, \dots, x_n)$ . These variables may or may not be independent. Problems such as the following are very interesting in themselves; but more important, their solutions often provide the basis for making statistical inferences. Let  $Y$  be a random variable that is defined by a function of  $X_1, X_2, \dots, X_n$ , say  $Y = u(X_1, X_2, \dots, X_n)$ . Once the p.d.f.  $f(x_1, x_2, \dots, x_n)$  is given, can we find the p.d.f. of  $Y$ ? In some of the preceding chapters, we have solved a few of these problems. Among them are the following two. If  $n = 1$  and if  $X_1$  is  $N(\mu, \sigma^2)$ , then  $Y = (X_1 - \mu)/\sigma$  is  $N(0, 1)$ . Let  $n$  be a positive integer and